Pipeline simulation by the method of characteristics for calculating the pressure pulsation of a high-pressure water plunger pump

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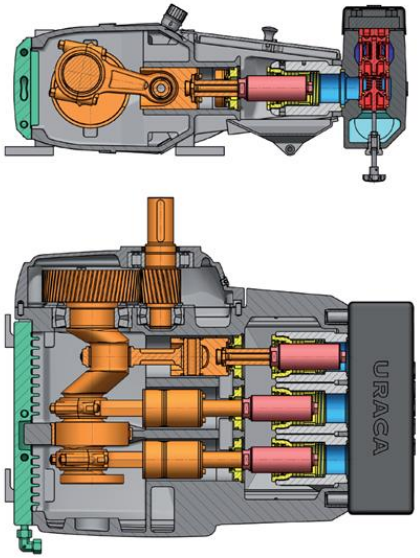
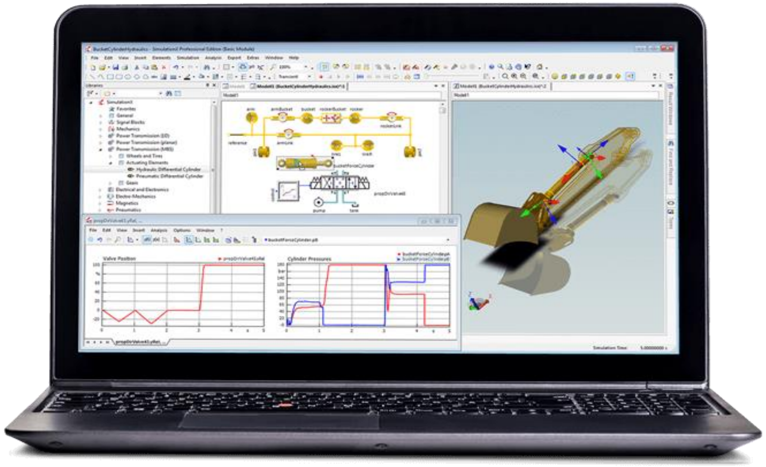
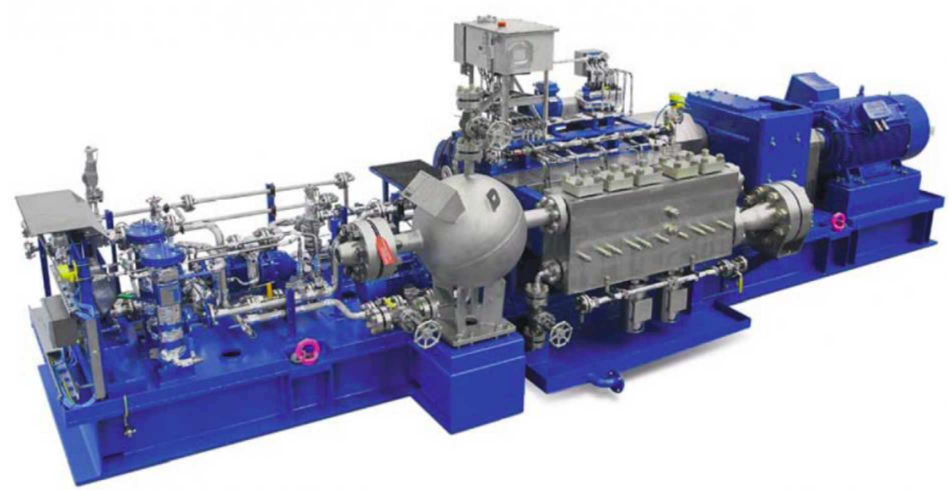


About us

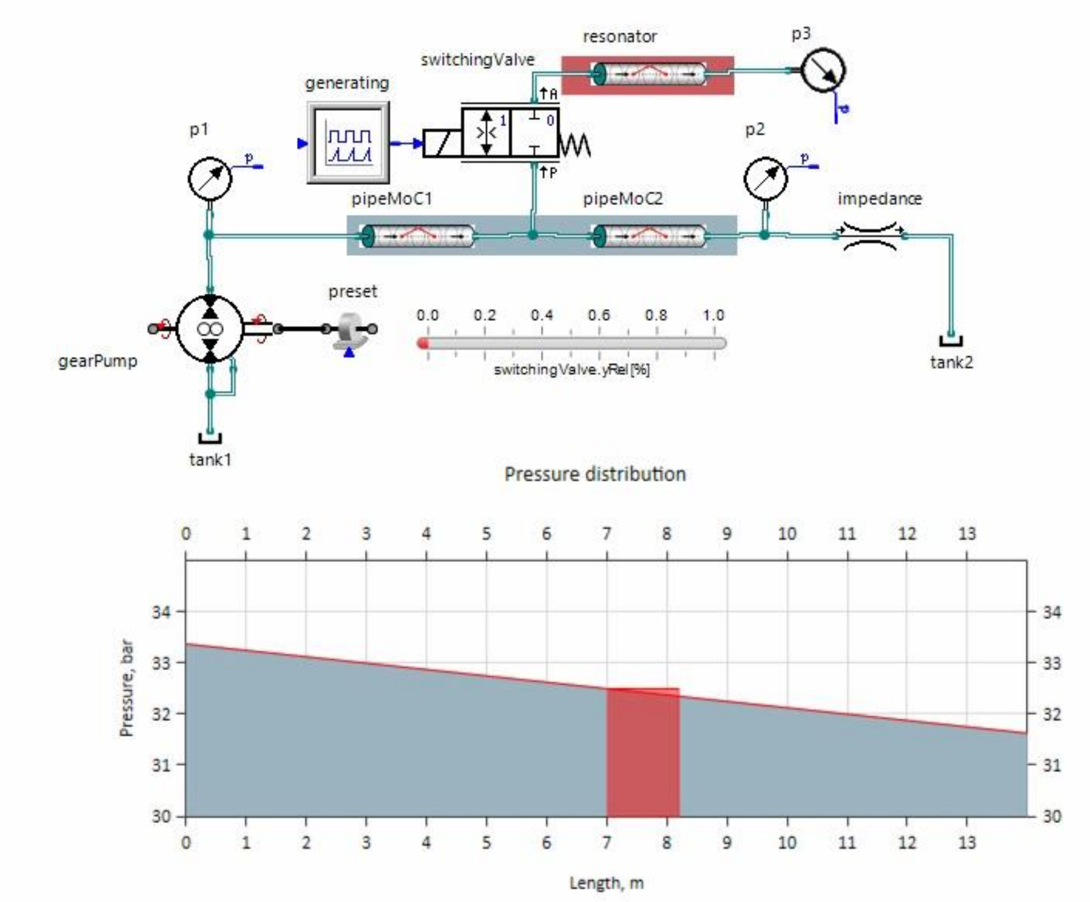




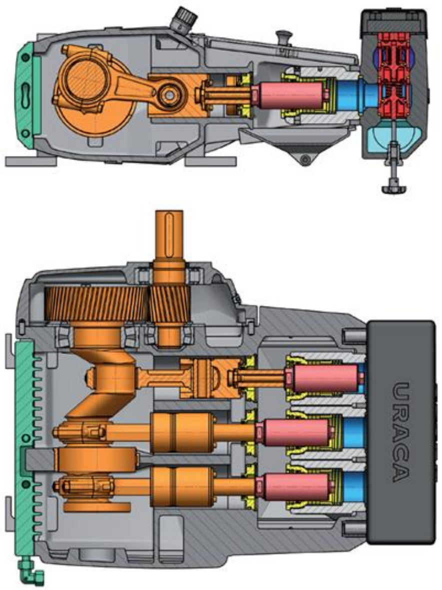
- **ESI ITI GmbH** – internationally operating engineering service provider and IT company
- Develops and sells the multiphysics simulation software **SimulationX**

- **URACA** was founded in 1893 and has been manufacturing pumps for over 100 years.
- Design and production of high pressure **plunger pumps** and pump units

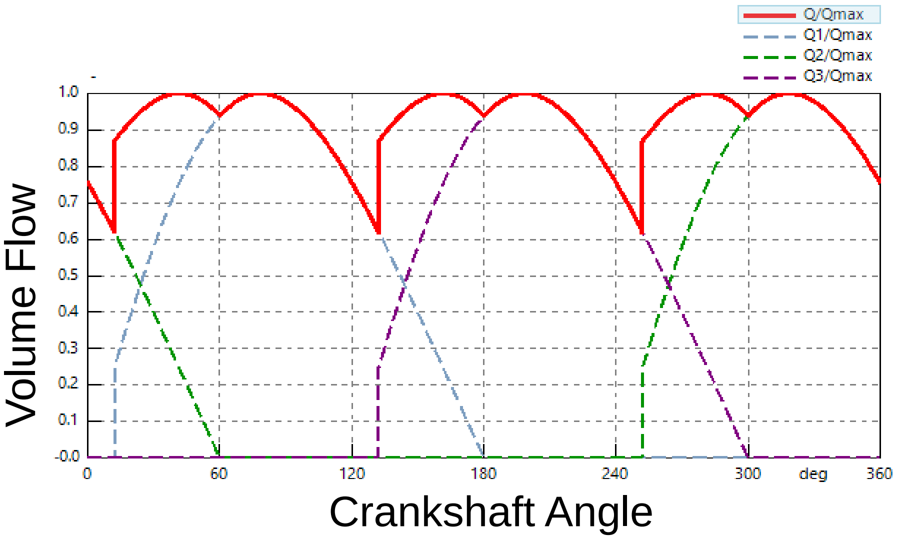
Pressure pulsation problem



Plunger pump as a source of pulsations



- Pump speed: up to 250 rpm
- Flow rate: up to 191 l/min
- Max. pressure: 250 bar



The following factors have a significant effect on the flow pulsation of the pump:

- A finite number of plungers
- Nonlinearity due to the kinematics of the mechanism
- Delays in the valve actuation due to the compressibility of the medium

One-dimensional model of wave propagation in pipelines

Assumptions:

- The pipeline is filled with liquid
- The cross section of the pipeline is constant and does not vary with pressure
- For calculations, the velocity averaged over the cross section of the pipe is used and the influence of the nonuniformity of the velocity distribution is neglected
- Heat exchange between liquid, pipe walls and the environment is not considered

Euler equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho v^2 + p)}{\partial x} = \rho\,(F + G)$$

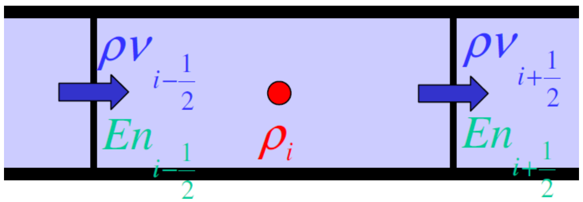
Euler equations in primitive variables:

$$\frac{\partial p}{\partial t} + \rho \cdot c^2 \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} = F + G$$

Methods of numerical solution

Finite control volume discretization

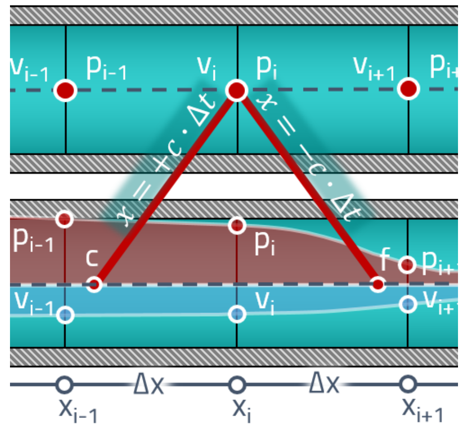


$$\frac{d\rho_i}{dt} = \frac{(\rho v)_{i-1} - (\rho v)_i}{\Delta x}$$

+ Ease of integration into an ODE-solver

- The numerical oscillations that arise when calculating the propagation of shock waves

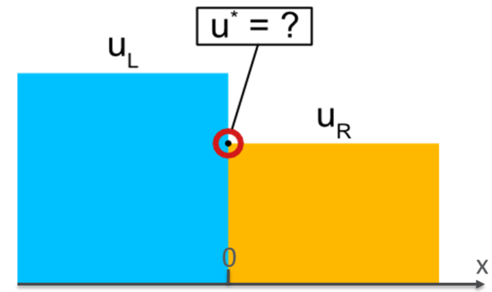
Method of characteristics



+ Allows for solving the problem of shock wave propagation in pipelines

- Restriction of the time step by the Courant–Friedrichs–Lewy condition.

Godunov‘s scheme

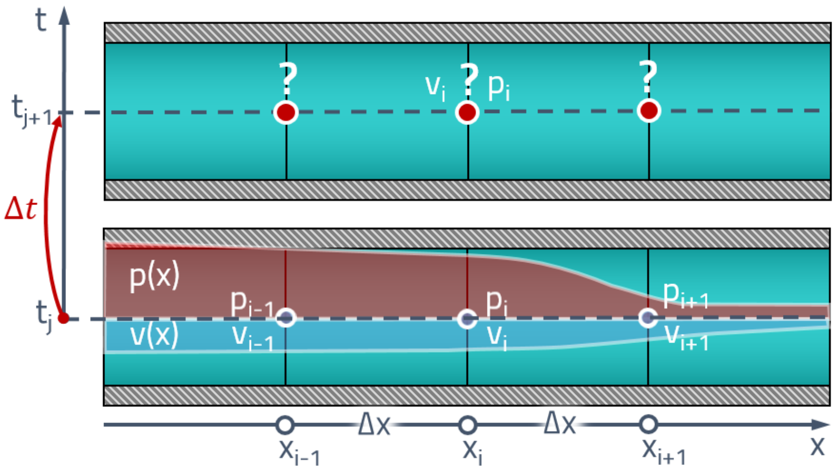


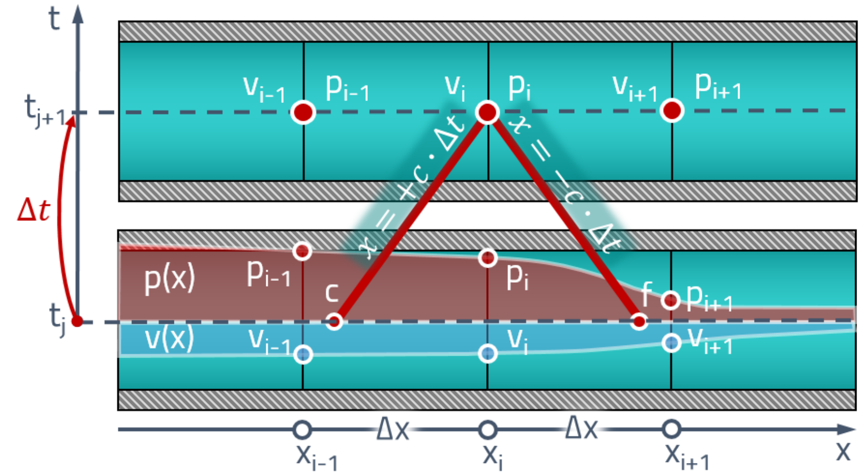
Riemann-problem

+ Allows for solving the problem of shock wave propagation of almost any complexity

- Usually more computationally time-consuming

Method of characteristics





- The method of characteristics discovers curves along which the PDE becomes an ODE
- Once the ODE is found, it can be solved along the characteristic curves and transformed into a solution for the original PDE

$$\left(v_{i,j+1} - v_c\right) + \frac{1}{\rho c}\left(p_{i,j+1} - p_c\right) - \left(F_c + G\right)\Delta t = 0$$

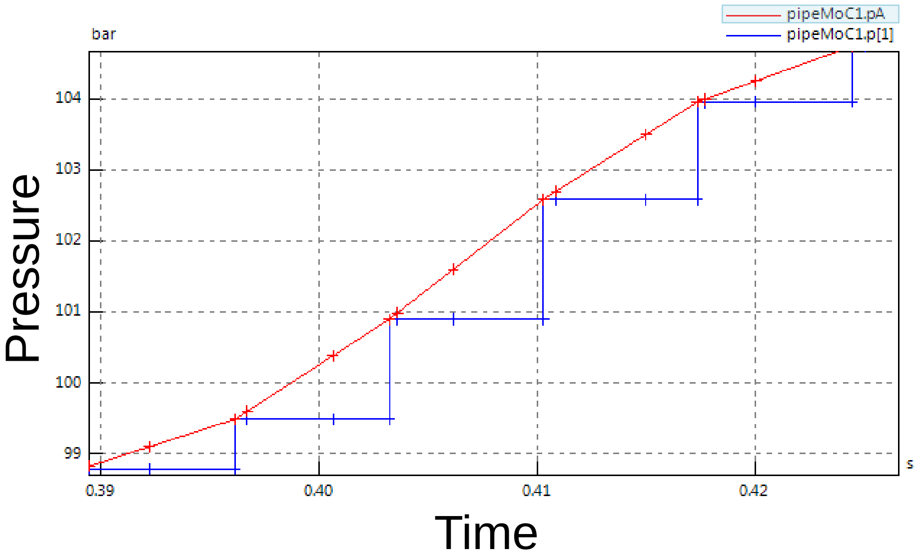
$$\left(v_{i,j+1} - v_f\right) - \frac{1}{\rho c}\left(p_{i,j+1} - p_f\right) - \left(F_f + G\right)\Delta t = 0$$

Method of characteristics

- The time step size is calculated by considering the Courant-Friedrich-Levy number (or Courant number) k_{CFL}:

$$\Delta t = k_{CFL} \frac{\Delta x}{c}$$

- For a stable numerical solution, choose the time step such that the $k_{CFL} < 1$.
- Pressure and velocity are calculated with a discrete time step.
- If the time step of ODE solver is smaller than the time step of the local solver of method of characteristics, the result will be in the form of steps.
- For smoothing the result, the pressures p_{Af}, p_{Bc} and velocities v_{Af}, v_{Bc} are calculated as follows:



- $\left(v_A - v_{Af}\right) - \frac{1}{\rho c}\left(p_A - p_{Af}\right) - \left(F_{Af} + G\right)\Delta t = 0$
- $\left(v_B - v_{Bc}\right) + \frac{1}{\rho c}\left(p_B - p_{Bc}\right) - \left(F_{Bc} + G\right)\Delta t = 0$
- $\frac{d}{dt}\left(u_{Af}\right) = \frac{u_{\{1\}f,j+1} - u_{\{1\}f,j}}{\Delta t}$
- $\frac{d}{dt}\left(u_{Bc}\right) = \frac{u_{\{n+1\}c,j+1} - u_{\{n+1\}c,j}}{\Delta t}$

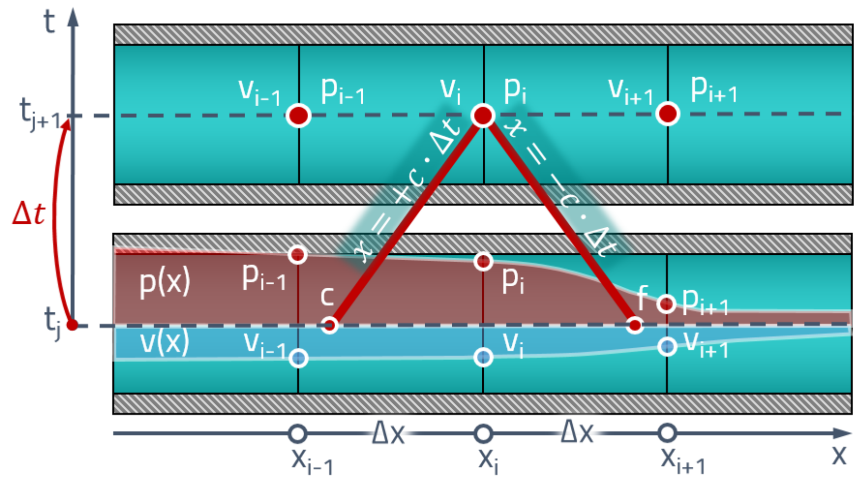
Method of characteristics

In case of variable density and speed of sound, the equations take the form:

- $\left(v_{i,j+1} - v_c\right) + \frac{1}{\overline{\rho_c} \cdot \overline{c_c}}\left(p_{i,j+1} - p_c\right) - \left(F_c + G\right)\Delta t = 0$
- $\left(v_{i,j+1} - v_f\right) - \frac{1}{\overline{\rho_f} \cdot \overline{c_f}}\left(p_{i,j+1} - p_f\right) - \left(F_f + G\right)\Delta t = 0$

where:

- $\overline{\rho_{c(f)}} = \frac{\rho_{c(f),j+1} + \rho_{c(f)}}{2}$
- $\overline{c_{c(f)}} = \frac{c_{c(f),j+1} + c_{c(f)}}{2}$

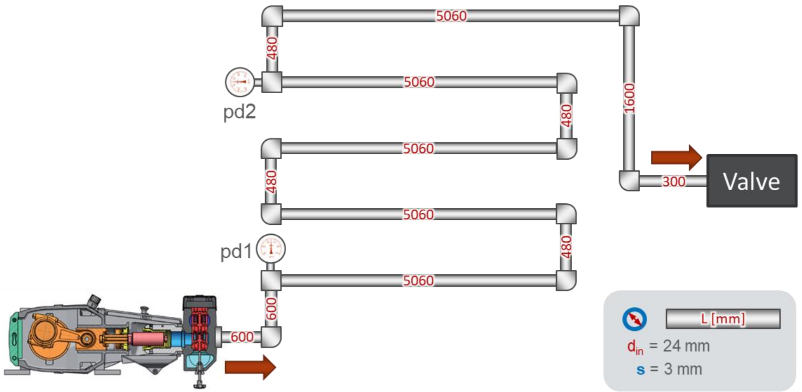


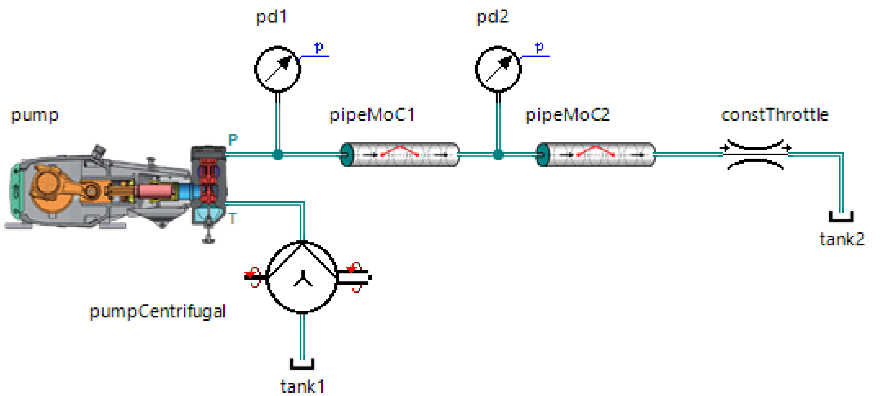
In these cases, the system of equations can be solved only by iterative methods. The following assumptions can significantly speed up the calculations:

- $\overline{\rho_{c(f)}} = \rho_M, \ \overline{c_{c(f)}} = c_M, \ \overline{v_{c(f)}} = v_M$

- $p_M = \frac{\sum_{i=1}^{n+1} p_i}{n+1}$
- $T_M = \frac{T_A + T_B}{2}$
- $\alpha_{UM} = \frac{\alpha_{UA} + \alpha_{UA}}{2}$

Test bench of URACA GmbH & Co. KG





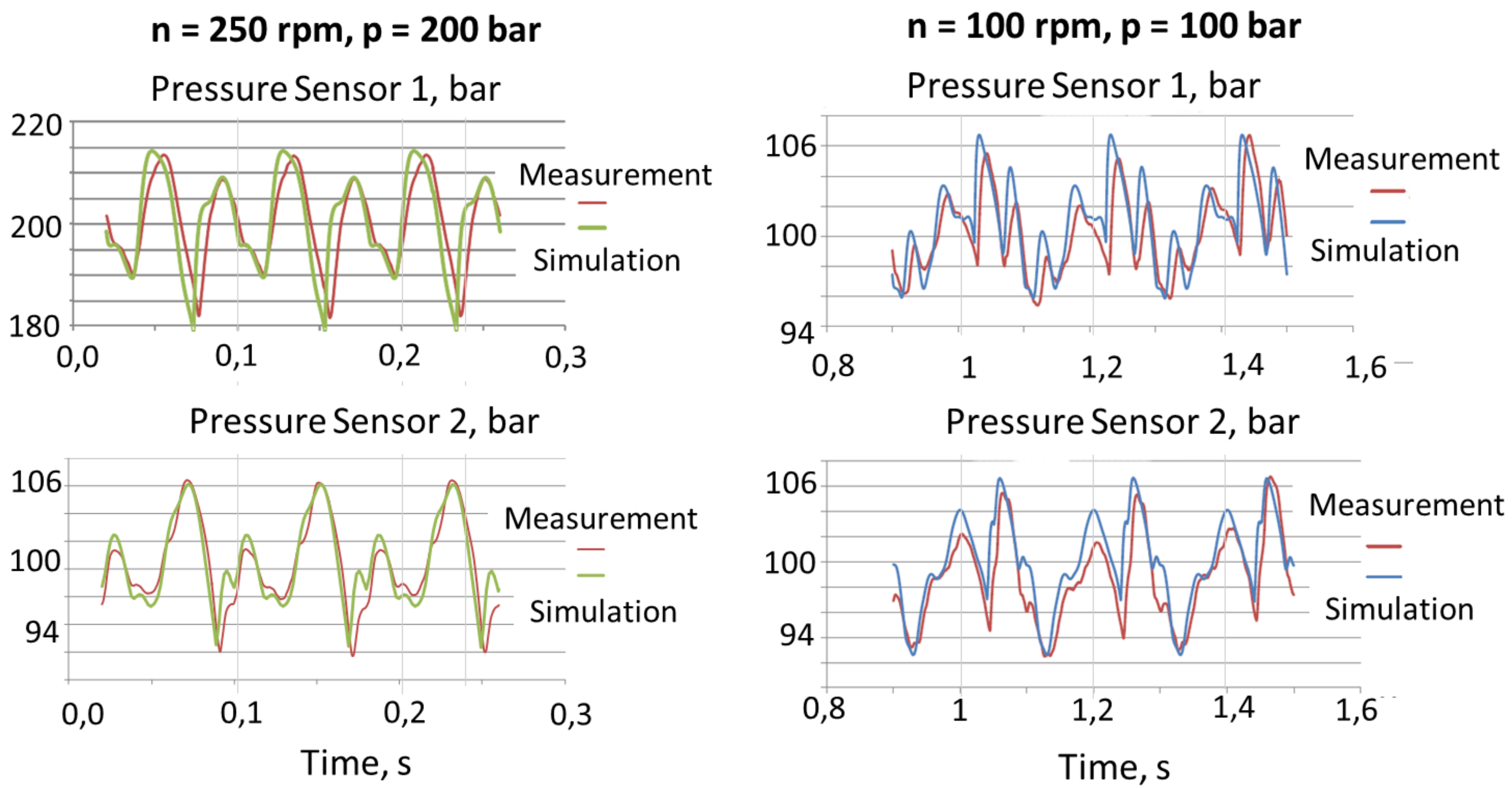
- The total length of the pipeline is 30 m. The purpose of the valve is to set the offset pressure.
- At the beginning of the pipeline, a pressure sensor pd1 is installed.
- The second pressure sensor pd2 is installed at 22 m from the sensor pd1.

In the plunger pump model:

- kinematics of the mechanism
- valve static characteristics
- compressibility of water in the working chamber.

The pipeline is simulated with two pipe models. Local resistance is not considered.

Simulation results



- Data shows a good correlation of the fundamental harmonics and characteristic peaks
- A significantly less damping in the simulation model can be observed

Conclusion

1. A high level of pulsations in the volume flow of a three-plunger pump makes it necessary to have a method for predicting the level of pressure ripples considering the pipeline.
2. The nonlinear nature of pulsations makes analysis in the time domain preferable.
3. To solve the problem of wave propagation in the pipeline, the method of characteristics was used with a local variable time step built into the ODE-solver.
4. Comparison with the results of the experiments showed the feasibility of using this model for preliminary calculations of pressure pulsations in the pipeline.
5. This model is effectively used by URACA GmbH&Co.KG to predict mechanical vibrations considering wave processes in the pipeline and to design resonance pulsation dampers.

Thank you for your attention!

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