Condition for Real-time Measurement of Power of Unsteady Fluid Flow in a Pipe by Kalman Filter

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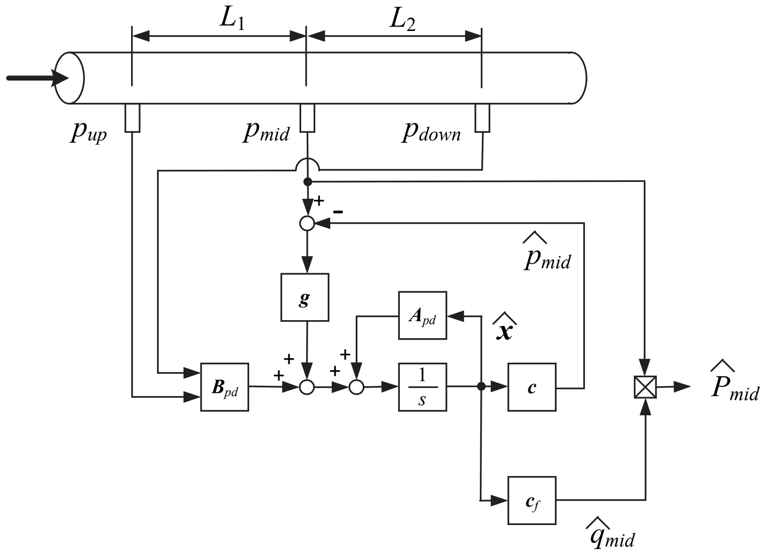

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3	Plant model of Kalman filter: Pipeline dynamics model
4	Off-line estimation of unsteady flow rate by Kalman filter
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1. Introduction

Electromagnetic flowmeter

Conductivity

http://www.keyence.co.jp/atsuryoku/ryuryou/fd_m/

Coriolis flow meter

U-shape tube

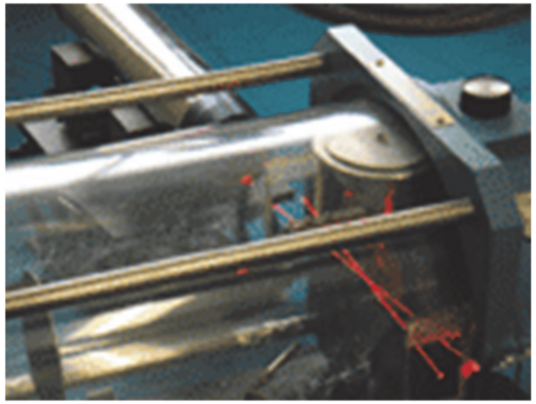
http://www.keyence.co.jp/atsuryoku/ryuryou/fd_s/

Ultrasonic flowmeter

http://www.fujielectric.co.jp/products/instruments/products/flow_ultra/FSV.html

Velocity distribution

Laser Doppler flow velocimeter



http://www.kanomax.co.jp/fsmart.html

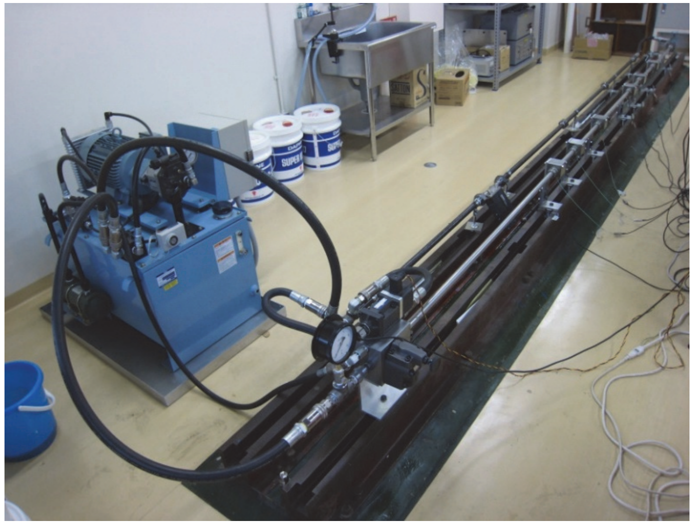
Pin-point Velocity

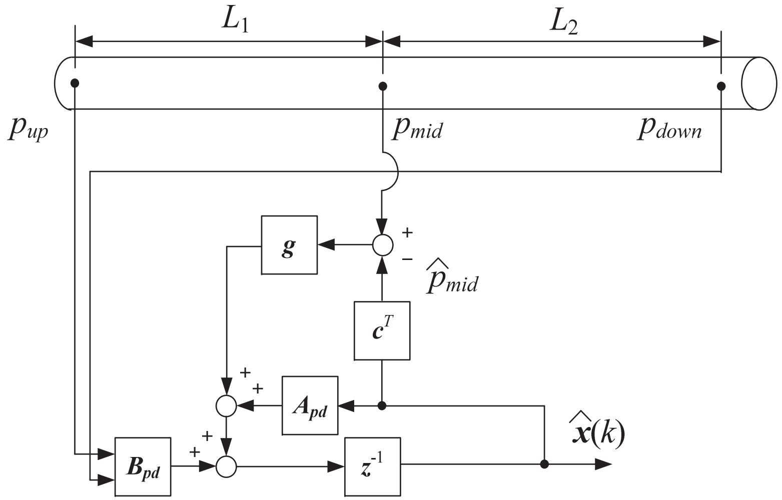
○Direct measurement in real-time

×sensor may disturb flow
×sensor is influenced by velocity distribution
×time-special distribution is not obtained

1. Introduction

- Kalman filter is proposed to estimate unsteady fluid flow in a pipe using a model of pipeline dynamics.
- The merit of this method is to use only three pressure sensors without flow sensor which may disturb flow in a pipe.
- The aim of this research is to develop a real-time indirect measurement system of unsteady flow rate and fluid power in a pipe.





2. Kalman filter theory

Kalman filter estimates the optimal state variables of a system which is disturbed by noise.

$$\boldsymbol{x_p}(k+1) = \boldsymbol{A}_{pd}\boldsymbol{x_p}(k) + \boldsymbol{B}_{pd}\overline{\boldsymbol{p}}(k) + \boldsymbol{B}_{pd}\boldsymbol{v}(k), \text{and}$$

$$p_{mid}(k) = \boldsymbol{c}^T\boldsymbol{x_p}(k) + w(k).$$

Priori estimation $\quad \hat{\boldsymbol{x}}^-(k) = \boldsymbol{A}_{pd}\hat{\boldsymbol{x}}(k-1) + \boldsymbol{B}_{pd}\overline{\boldsymbol{p}}(k-1), \text{and}$

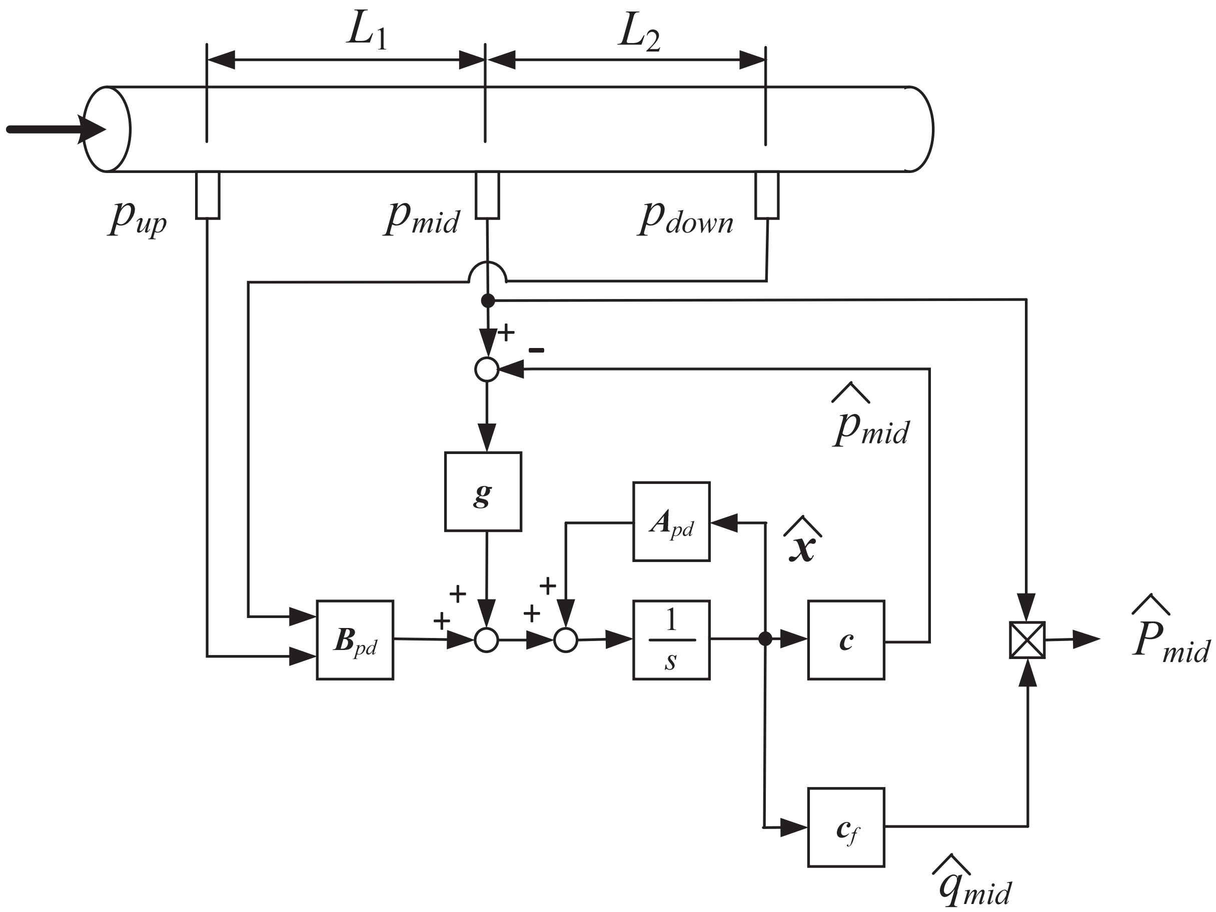
Posteriori estimation $\quad \hat{\boldsymbol{x}}^-(k) = \boldsymbol{A}_{pd}\hat{\boldsymbol{x}}(k-1) + \boldsymbol{B}_{pd}\overline{\boldsymbol{p}}(k-1)$

Kalman gain $\quad \boldsymbol{g} = \dfrac{\boldsymbol{Pc}}{\boldsymbol{c}^T\boldsymbol{Pc} + \sigma_w^2}$

Riccati equation $\quad \boldsymbol{P} = \boldsymbol{A}_{pd}\left[\boldsymbol{P} - \dfrac{\boldsymbol{Pcc}^T\boldsymbol{P}}{\boldsymbol{c}^T\boldsymbol{Pc} + \sigma_w^2}\right]\boldsymbol{A}_{pd}^T + \boldsymbol{\sigma}_v^2\boldsymbol{B}_{pd}\boldsymbol{B}_{pd}^T,$

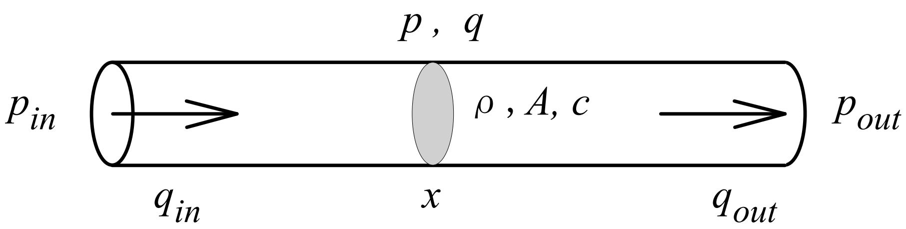
Variances $\quad \sigma_w^2 = E(w^2), \text{and} \quad \boldsymbol{\sigma}_v^2 = E(\boldsymbol{v}\boldsymbol{v}^T).$

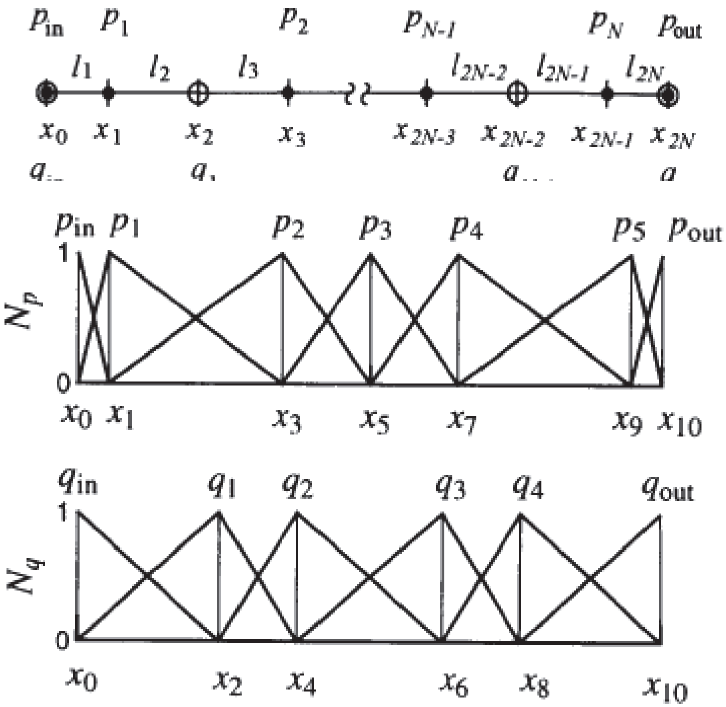
2. Kalman filter theory



A measurement system using a Kalman filter

3. Pipeline mode; Optimized finite element model of pipeline dynamics





Interlacing grid system and linear shape function

Equation of motion

$$\frac{\partial q}{\partial t} + \frac{A}{\rho}\frac{\partial p}{\partial x} + p_f(q) = 0$$

Continuity equation

$$\frac{\partial p}{\partial t} + \frac{\rho c^2}{A}\frac{\partial q}{\partial x} = 0$$

Finite element approximation

$$\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} + \frac{A}{\rho}\boldsymbol{B}_{ofem}\,\boldsymbol{q} + \frac{A}{\rho}\boldsymbol{F}_{ofem}\,\overline{\boldsymbol{p}} + \boldsymbol{p}_f = \boldsymbol{0}$$

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} + \frac{\rho c^2}{A}\boldsymbol{E}_{ofem}\,\boldsymbol{q} = \boldsymbol{0}$$

State variables at grid points

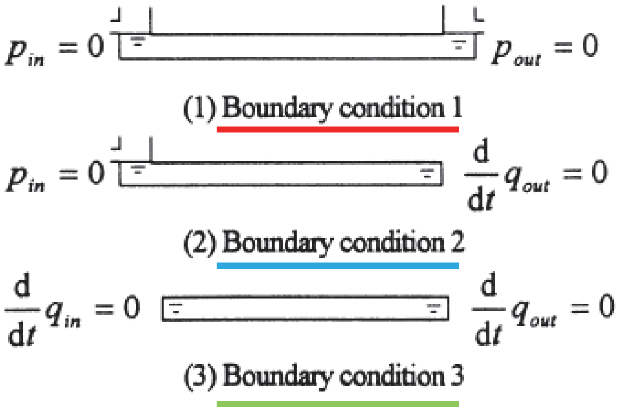
$$\boldsymbol{q} = \left[q_U, q_1, \ldots q_{N-1}, q_D\right]^T$$

$$\boldsymbol{p} = \left[p_1, p_2 \ldots p_N\right]^T$$

3. Pipeline model; Optimized finite element model of pipeline dynamics

The interlacing grid spacing is optimized to minimize the error function of model’s eigenvalues compared with theoretical ones.

$$J(l_1, l_2, \cdots l_{N-1}) = \sum_{i=1}^{k_1}\left(\frac{\omega_{1i}}{2i\omega_n}-1\right)^2 + \sum_{i=1}^{k_2}\left(\frac{\omega_{2i}}{(2i-1)\omega_n}-1\right)^2 + \sum_{i=1}^{k_3}\left(\frac{\omega_{3i}}{2(i-1)\omega_n}-1\right)^2$$

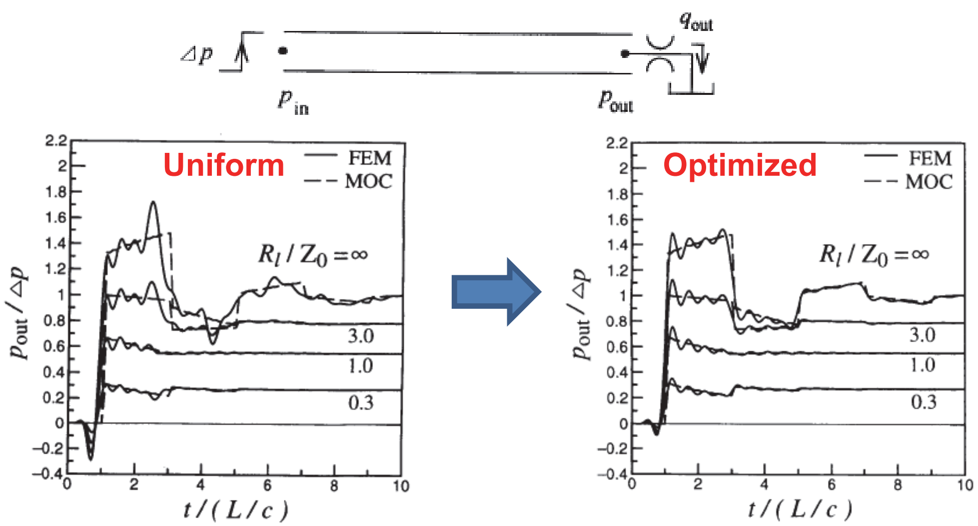


$2i\omega_n$

$(2i-1)\omega_n$

$2(i-1)\omega_n$

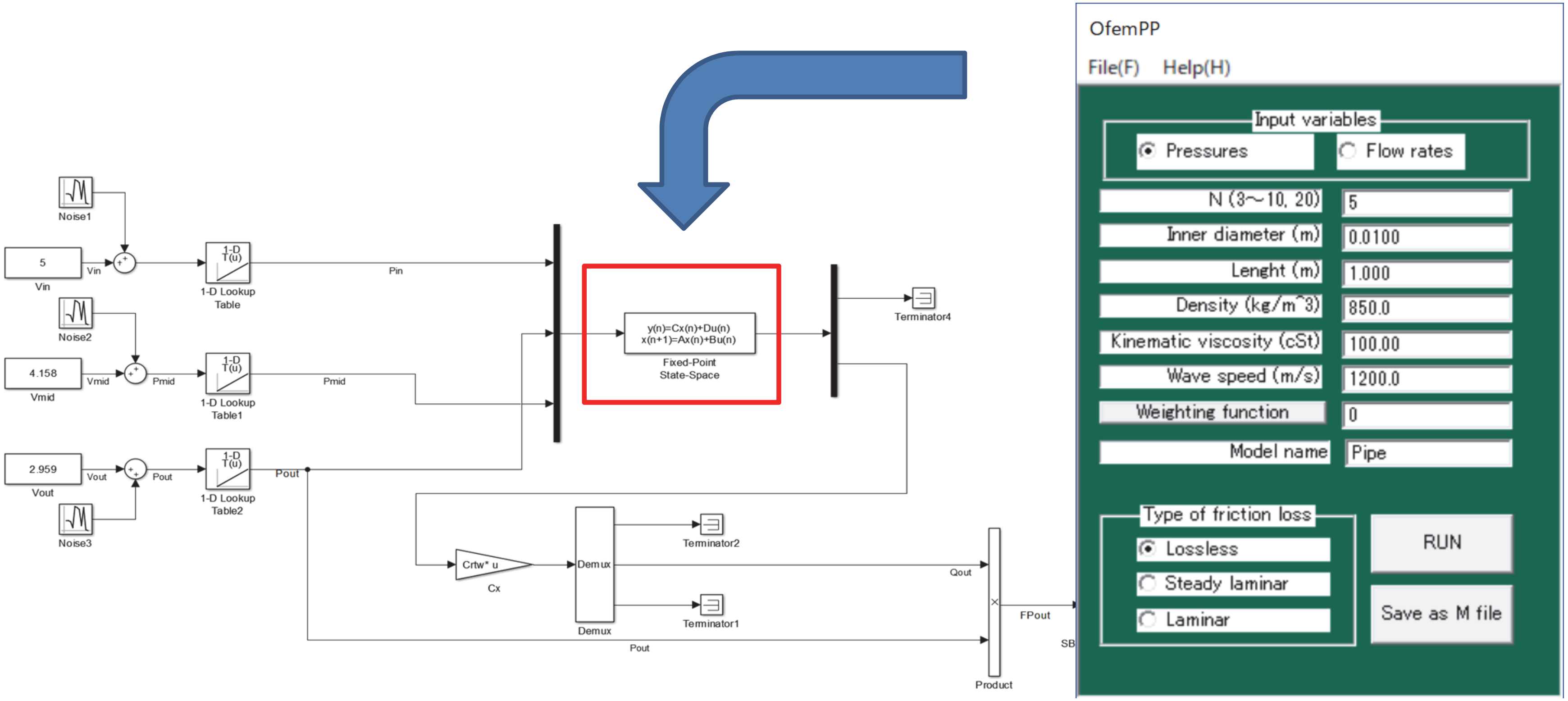
$$\omega_n = \frac{\pi c}{2L}$$



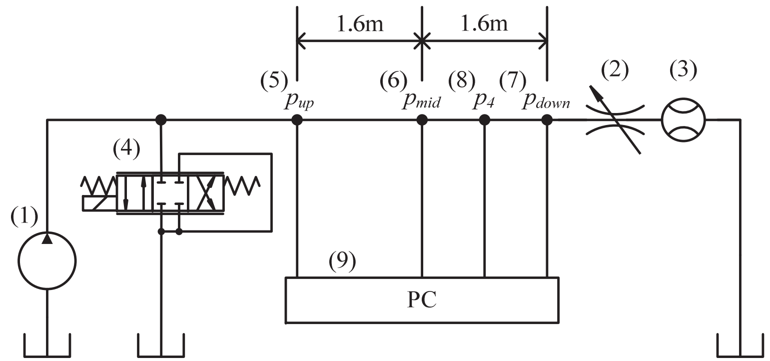
3. Pipeline model; Optimized finite element model of pipeline dynamics

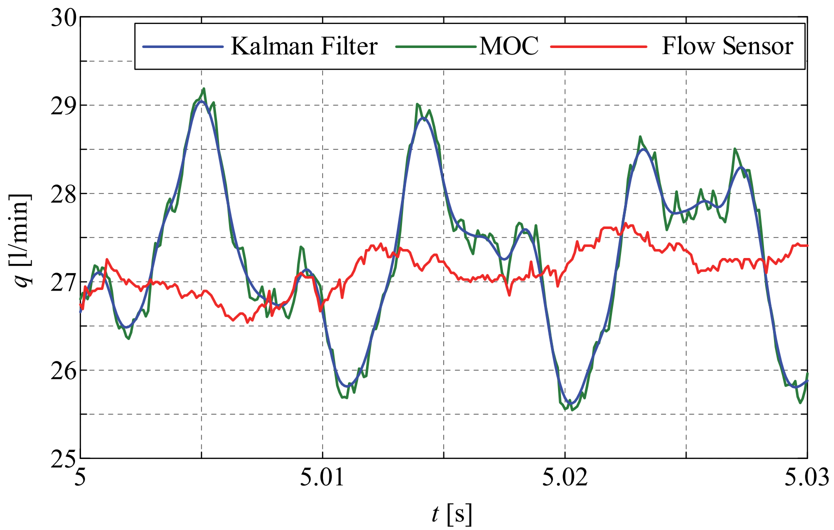
Optimized finite element model can be used by a state space block of MATLAB/Simulink.

GUI for automatic calculation of state space representation

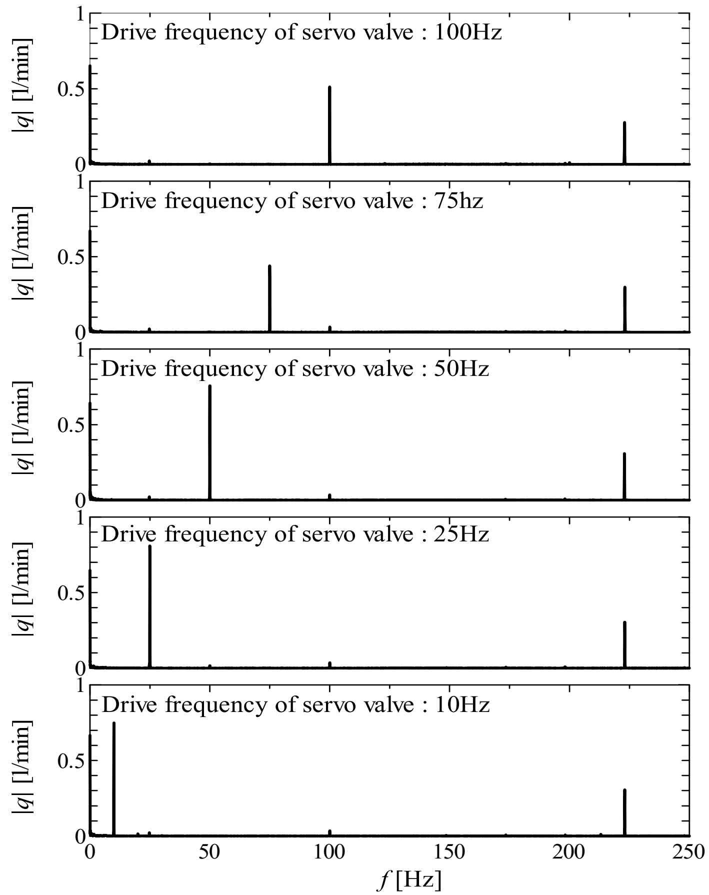


4. Off-line estimation of unsteady flow rate by the Kalman filter





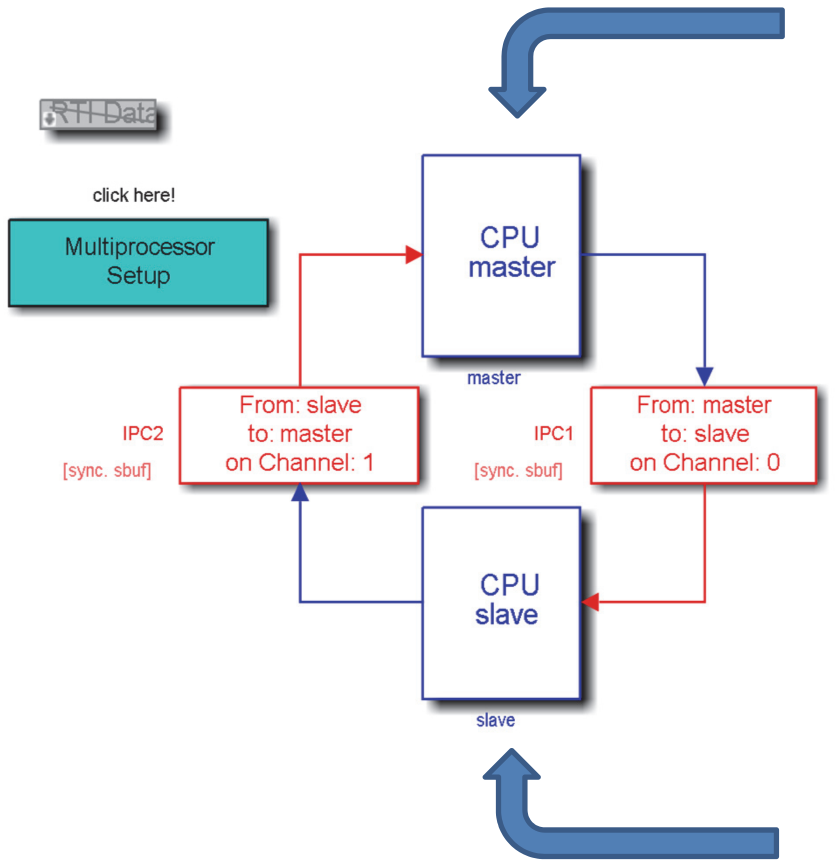
A test circuit of measuring unsteady flow rate

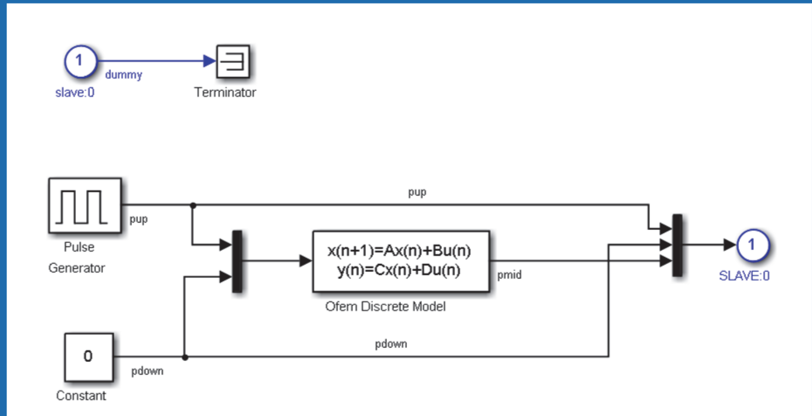


FFT analysis of unsteady flow rate estimated by the Kalman filter

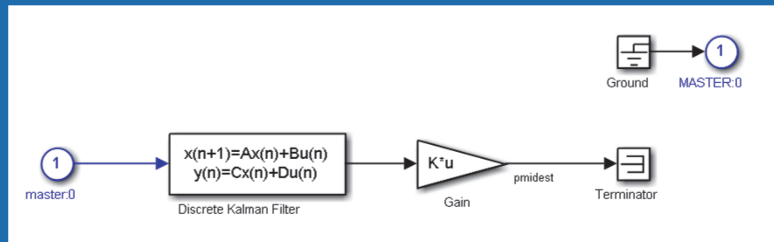
5. Real-time implementation of Kalman filter

Kalman filter is installed in a real-time computing system.



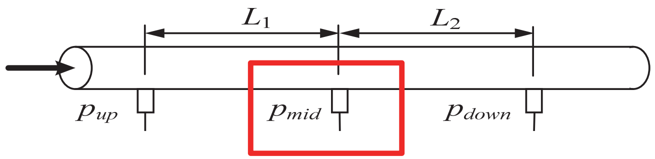


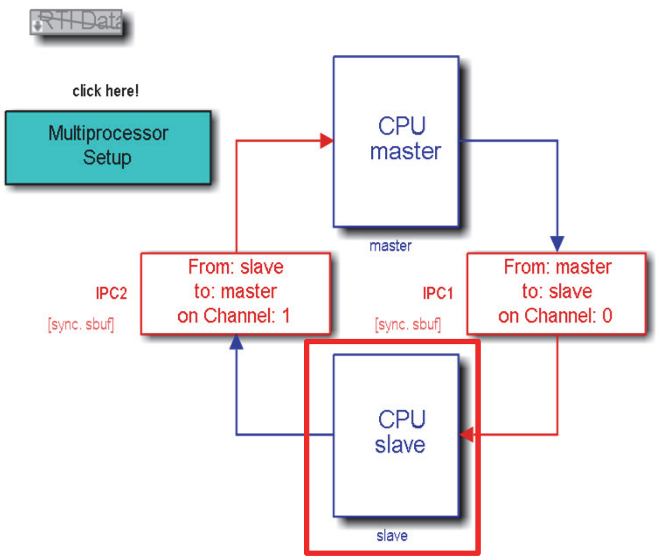
Target pipe simulator

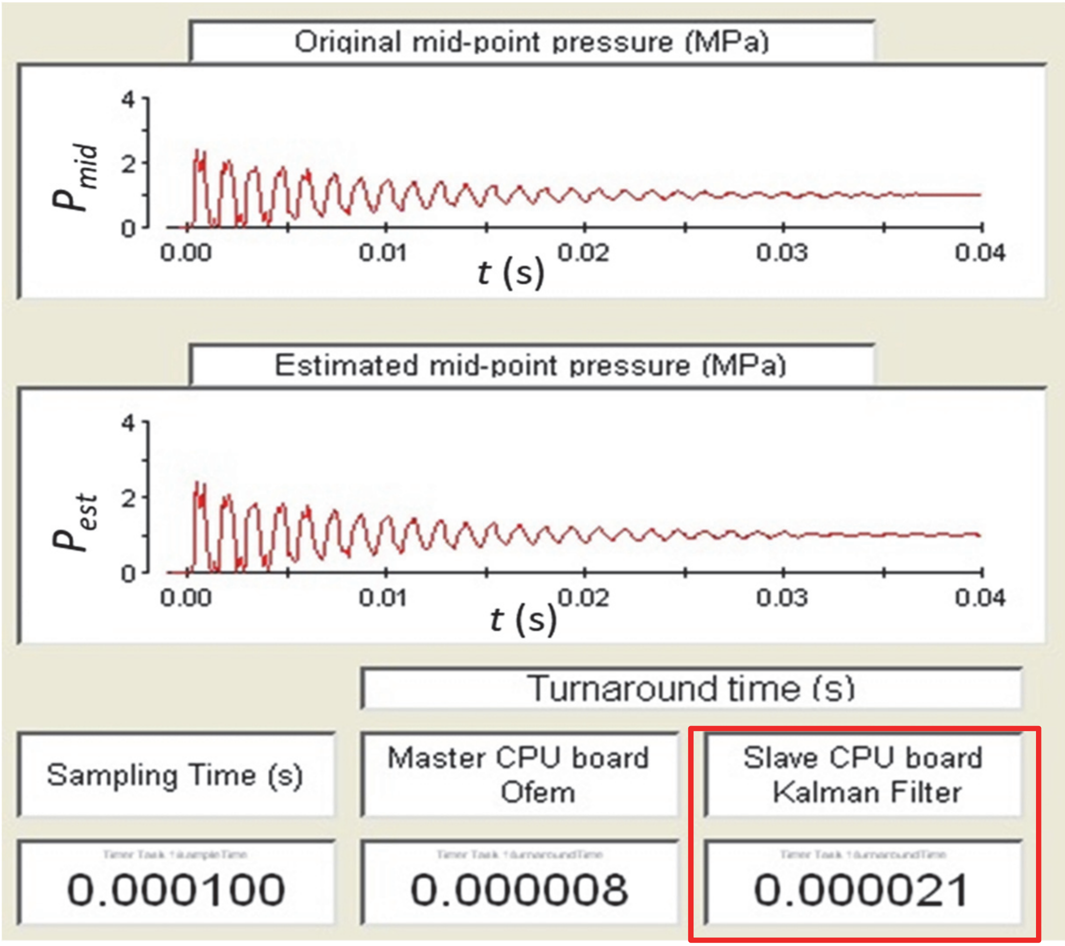


Kalman filter

5. Real-time implementation of Kalman filter







100μs > 21μs

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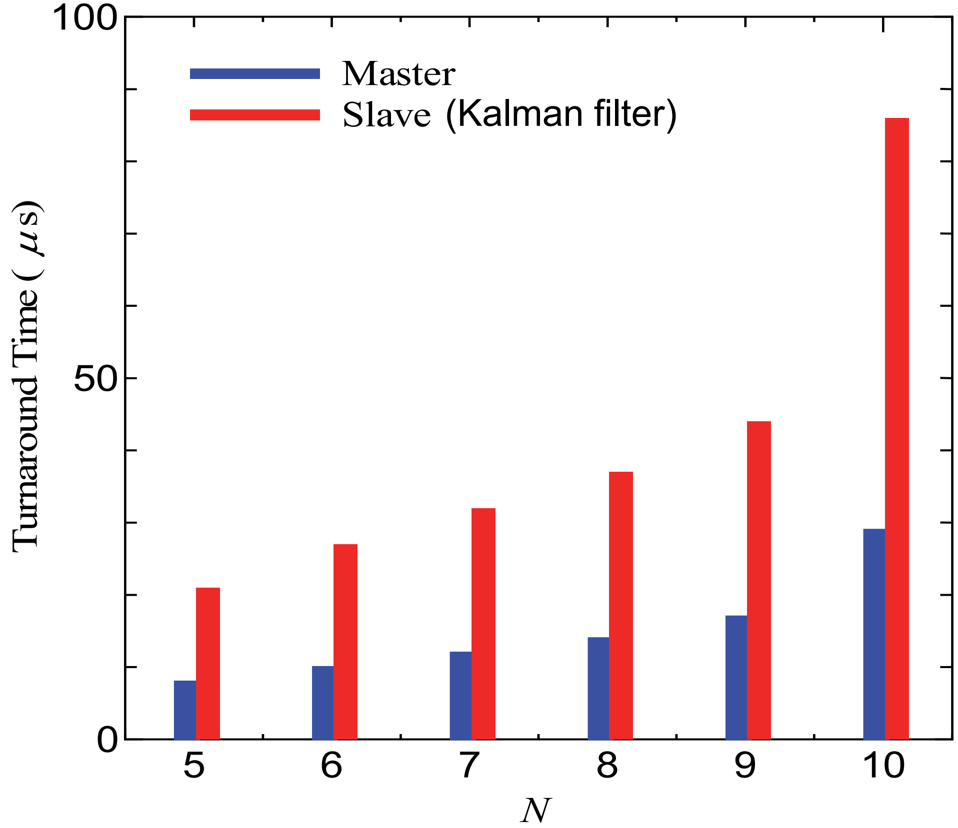
5. Real-time implementation of Kalman filter



Turnaround time for various numbers of elements N
(Sampling time: 100μs for N=9 or less, 1000μs for N=10)

CPU elapsed time depends on the number of finite elements of pipeline model.

6. Conclusion

- Real-time implementation of the Kalman filter is proposed to measure incompressible unsteady laminar flow rate and power of fluid flow in a pipe.
- A steady-state Kalman filter is implemented in a real-time computing system.
- It successfully works with a sampling time of 100 μs.

Future works

- to reduce computational task
- to reduce the order of the pipeline model by order-reduction theory, etc.
- to select suitable number of finite elements

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Thank you for your attention!

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