



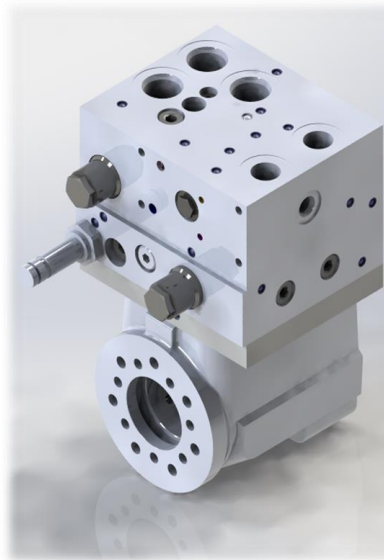
Identification and synthesis of linear-quadratic regulator for digital control of electrohydraulic steering system

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- 1 Introduction
- 2 Experimental system layout
- 3 Multivariable system identification
- 4 Design of linear-quadratic regulator
- 5 Experimental results
- 6 Conclusion

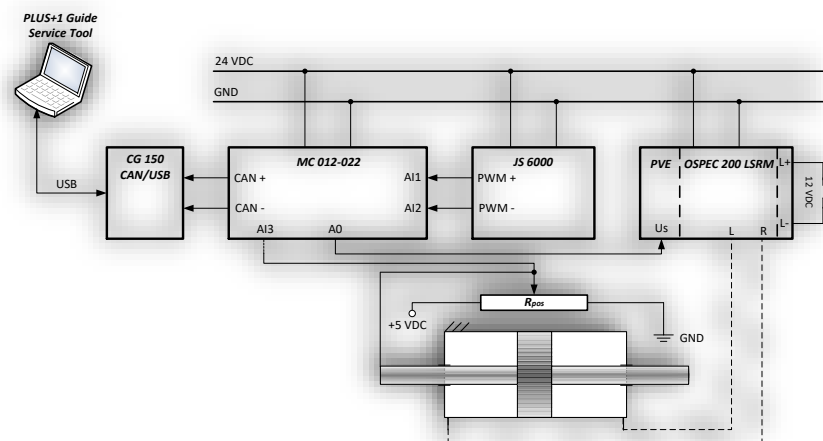
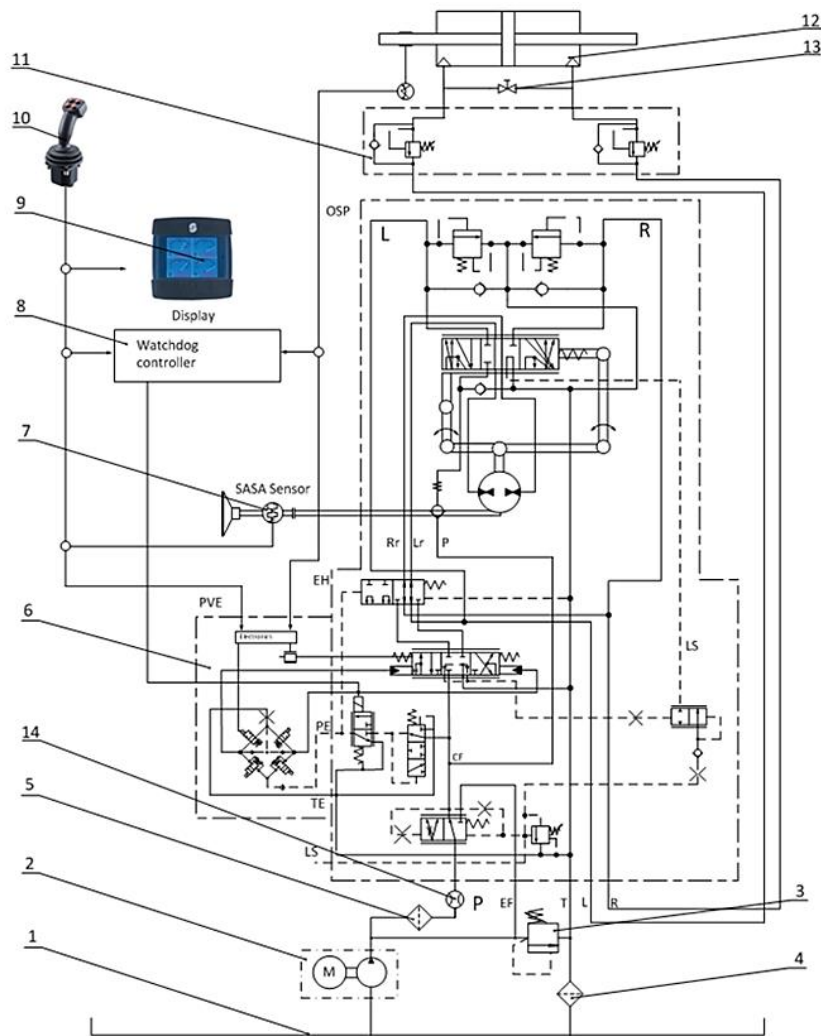
- ❖ In modern mobile machines, the proportional electrical control of the steering system is used due to the need for remote control via GPS. In addition, the electrohydraulic steering with variable steering ratio from the steering wheel to the machine's steer axle is often a sought after function to improve driver productivity and comfort. This leads to the need for an effective integrated control system which should ensure the quality behavior of the entire electrohydraulic system.
- ❖ Steering control systems are necessary because loading torques acting upon steering axle may disturb steering performance. Also the mathematical model cannot take into account all physical details because it would become impractically complex.
- ❖ The reason is not only the intuitive idea behind PID but also because it has proved robust to small model uncertainties. However PID controller tuning becomes a difficult task in case of many inputs many outputs (MIMO) plant. For MIMO case the control theory suggests many advanced control techniques, which can take into account multivariable nature of the process. Such practical approved control technique is linear quadratic Gaussian regulator (LQG) that involves linear quadratic regulator (LQR) and uses state estimates obtained by Kalman filter. The LQG algorithm takes into account not only multivariable process nature but also the influence of noises to the plant dynamics.



Main objective:

- The main objective of this work is to present the designed system for control of electrohydraulic steering system that is implemented in low speed mobile machines.
- The goal of control algorithm is to achieve fast transient response without overshooting and static error in whole working range. To achieve this aim first a multivariable dynamical plant model is estimated by identification procedure. The model obtained is validated by various statistical tests.
- The multivariable **LQR** regulator with integral action and Kalman filter are designed.
- Appropriate software which is implemented in 32-bit microcontroller is developed. Experimental results are presented which confirm that the control system achieves the prescribed performance.

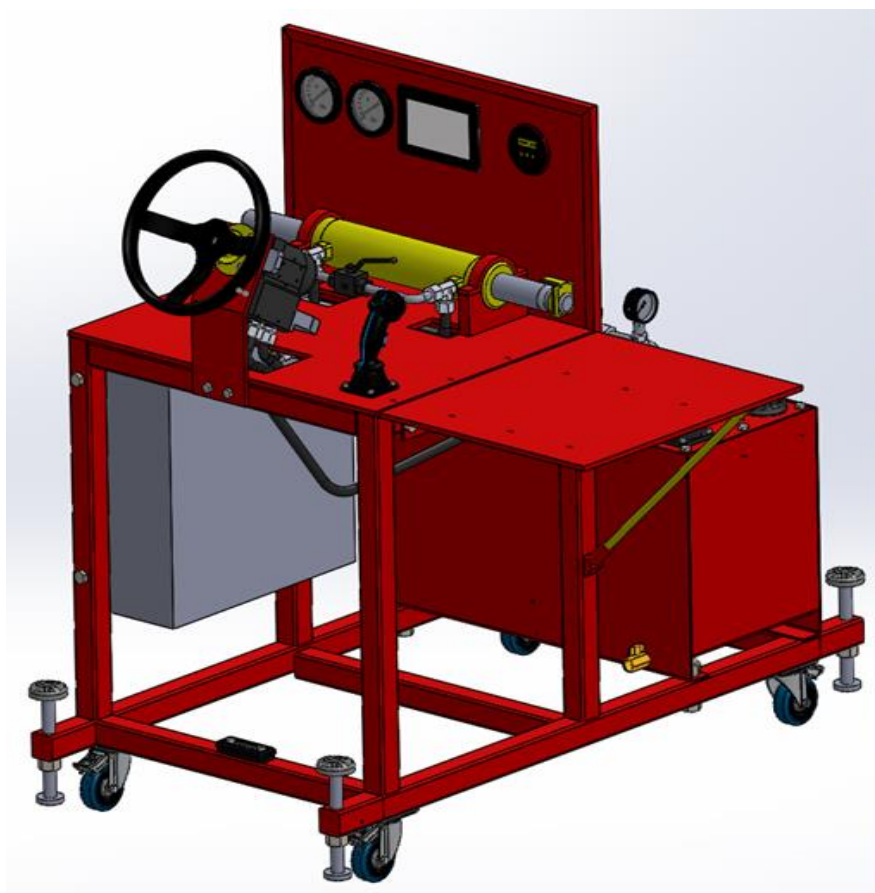
Experimental system layout



No	Components	Parameters
2.	Motor-pump group	$P_{em}=7.5$ kW; $V_p=19$ cm ³
3.	Pressure-relief valve	$q_{nom}=40$ l/min; $\Delta p_{nom}=25$ MPa
5.	Pressure filter	$\eta=10$ μm
6.	EH Steering unit	$V_p=200$ cm ³
7.	Steering wheel sensor	CAN-output
8.	Microcontroller	32-bit ADC;PWM;
10.	Joystick	y-coordinate; CAN
11.	Over-center valves	$q_{nom}=50$ l/min $\Delta p_{nom}=20$ MPa
12.	Servo-cylinder	80x50x300 mm
14.	Flow rate meter	$q_{nom}=50$ l/min

Experimental system layout

3D model

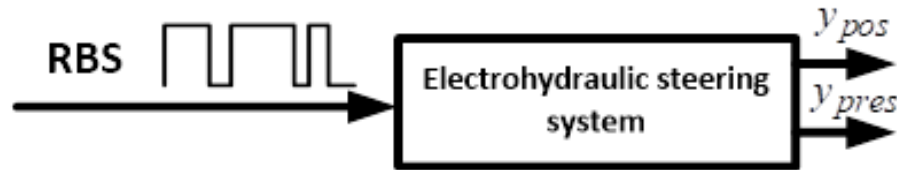


Real implementation



Multivariable system identification

Thus the goal of identification is to obtain a linear black box model which sufficiently well describes electrohydraulic steering system dynamics and noises in wide working range.



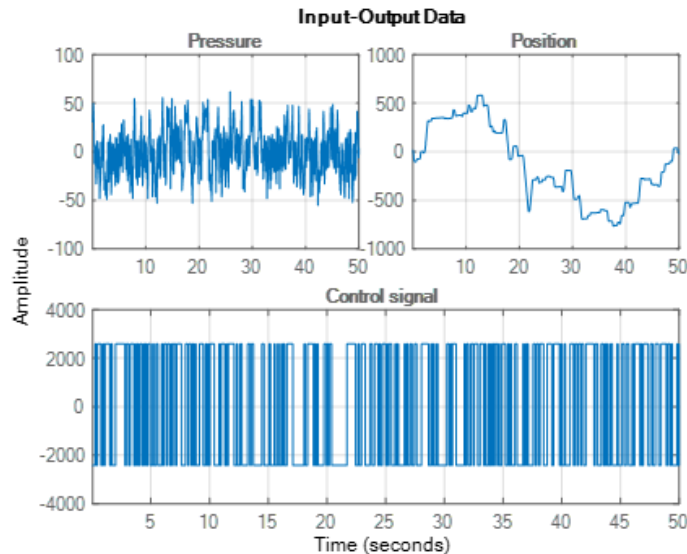
- To obtain such model first the open loop identification experiments is designed.
- The sample time of $T_s = 0.05s$ is chosen, that is sufficiently small.
- Plant input a random binary signal (RBS) is applied.
- The amplitude of RBS is chosen to be ± 2500 .
- The excitation level of identification input signal is 500. This means that up to 500 parameters can be estimated from estimation data set.
- The identification procedure starts with estimation of state space model with free parameterization

$$x(k+1) = Ax(k) + Bu(k) + K_v v(k)$$

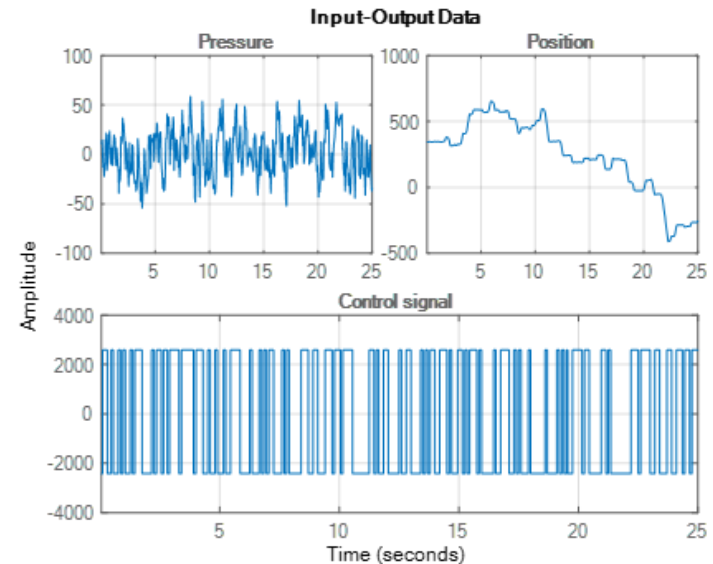
$$y(k) = Cx(k) + Du(k) + v(k)$$

where $x(k) = [x_1 \ x_2 \ \dots \ x_n]^T$ is a state vector, $u(k)$ is the input signal, $y(k) = [y_{pres} \ y_{pos}]^T$ is the output vector, $v(k)$ is a model disturbance (residual) and A, B, C, D, K_v are the matrices with appropriate dimensions.

Estimation data set



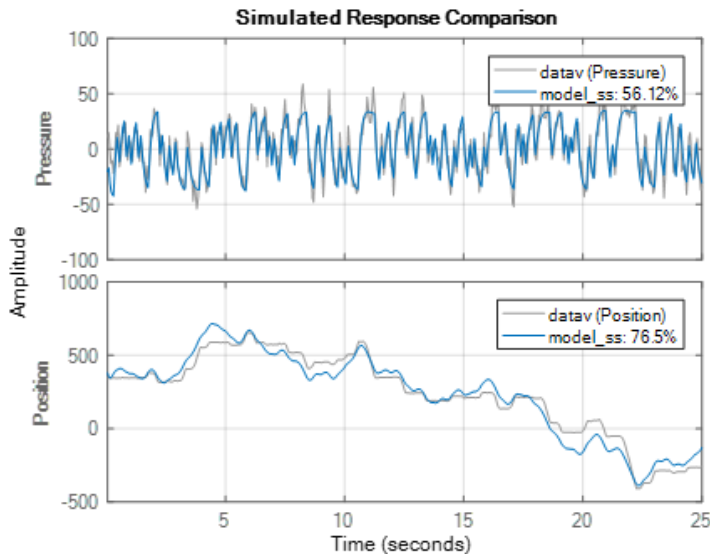
Validation data set



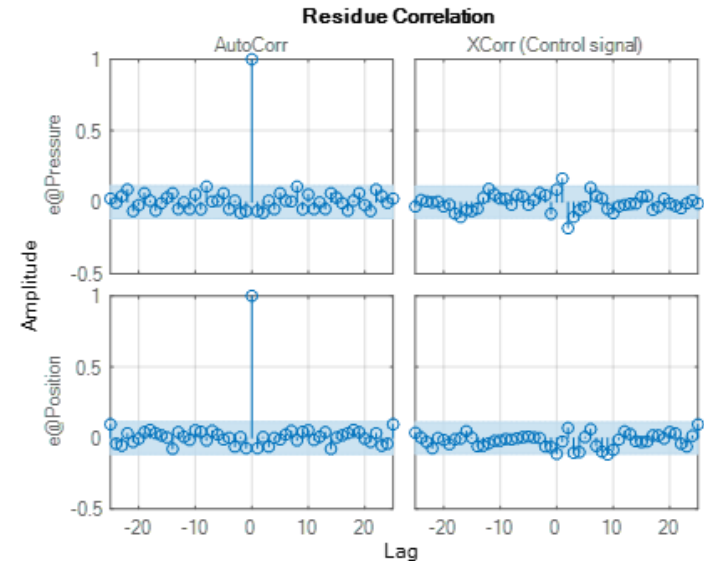
It is chosen to estimate a state space model because it has a form which can be directly used for Kalman filter design with MATLAB. After estimation of these models by prediction error method, the validation tests are performed. The model of order 3 is chosen, because it is a simplest model from model set that passes the validation procedure.

$$A = \begin{bmatrix} 0.8769 & -0.3987 & 0.3986 \\ 0 & 0 & 1 \\ -0.1666 & -0.5099 & 1.509 \end{bmatrix}, B = \begin{bmatrix} -0.0043 \\ 0.0011 \\ 0.0021 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, K_v = \begin{bmatrix} 0.1112 & -0.06214 \\ -0.09525 & 1.55 \\ -0.2003 & 1.897 \end{bmatrix}$$

Model and measured outputs



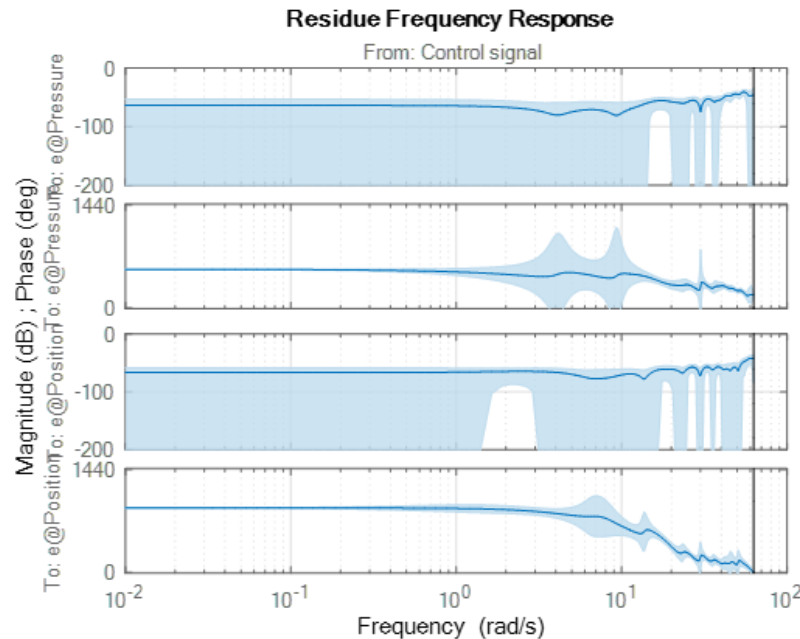
Residual test



The value of FIT between measured pressure and model pressure is 56.12% and the one between the measured position and model position is 76.5%. These results mean that estimated model captures sufficiently well plant dynamics.

As can be seen the noise model is adequate and there is not significant correlation between input and residuals.

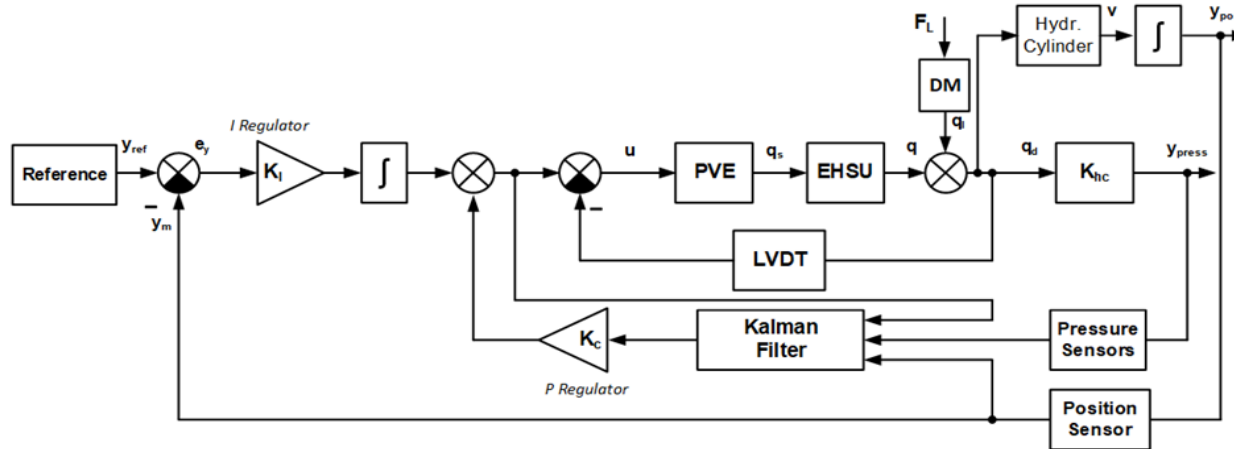
Residuals to input signal frequency response



The frequency response of estimated high order finite impulse response (FIR) model between control signal and residuals along with 99% confidence region. As a result from identification a 3-th order state space model is obtained. It describes sufficiently well the both a plant and a noise dynamics. The plant dynamic model will be used for LQR controller design whereas the noise model will be used for Kalman filter design.

Design of linear-quadratic regulator

LQR with Kalman filtering closed-loop schematic



To ensure sufficiently well reference tracking an **LQR** controller with integral action is designed. The design is done on the basis of deterministic part of estimated model which is extended with an extra state. This extra state is discrete time integral of position error

$$x_i(k+1) = x_i(k) + T_s e_y(k) = x_i(k) + T_s (y_{ref}(k) - y_{pos}(k))$$

where $y_{ref}(k)$ is the reference. Thus, combining the deterministic part of equation and equation one obtains the augmented system

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}u(k) + \bar{G}y_{ref}(k),$$

$$y(k) = \bar{C}\bar{x}(k),$$

Design of linear-quadratic regulator

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ x_i(k) \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 \\ -T_s C & 1 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \bar{G} = \begin{bmatrix} 0 \\ T_s \end{bmatrix}.$$

The optimal control law is obtained in the form

$$u(k) = -\bar{K}\bar{x}(k), \bar{K} = [K_c \quad -K_i]$$

where K_c is the proportional term matrix gain and K_i is the integral term gain. The controller matrix \bar{K} is obtained from minimization of quadratic performance index

$$J(u) = \sum_{k=0}^{\infty} \bar{x}^T(k) Q \bar{x}(k) + u^T(k) R u(k)$$

where Q and R are positive definite matrices chosen to ensure acceptable transient response of the closed-loop system. The optimal feedback matrix \bar{K} is determined by

$$\bar{K} = (R + B^T P B)^{-1} B^T P A$$

where P is the positive definite solution of the discrete-time matrix algebraic Riccati equation

$$A^T P A - P - A^T P B (R + B^T P B)^{-1} B^T P A + Q = 0$$

Design of linear-quadratic regulator

The optimal controller matrix is obtained for $Q = \begin{bmatrix} 7 \times 10^4 & 0 & 0 & 0 \\ 0 & 10^4 & 0 & 0 \\ 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Since the state $x(k)$ of system is not accessible, the optimal control law is implemented as

$$u(k) = -K_c \hat{x}(k) + K_i x_i(k)$$

where $\hat{x}(k)$ is estimate of $x(k)$. It is obtained by discrete time Kalman filter

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K_f (y(k+1) - CBu(k) - CA\hat{x}(k))$$

The filter matrix K_f is determined as

$$K_f = D_f C^T (CDC^T + 10^{-4} I_2)^{-1}$$

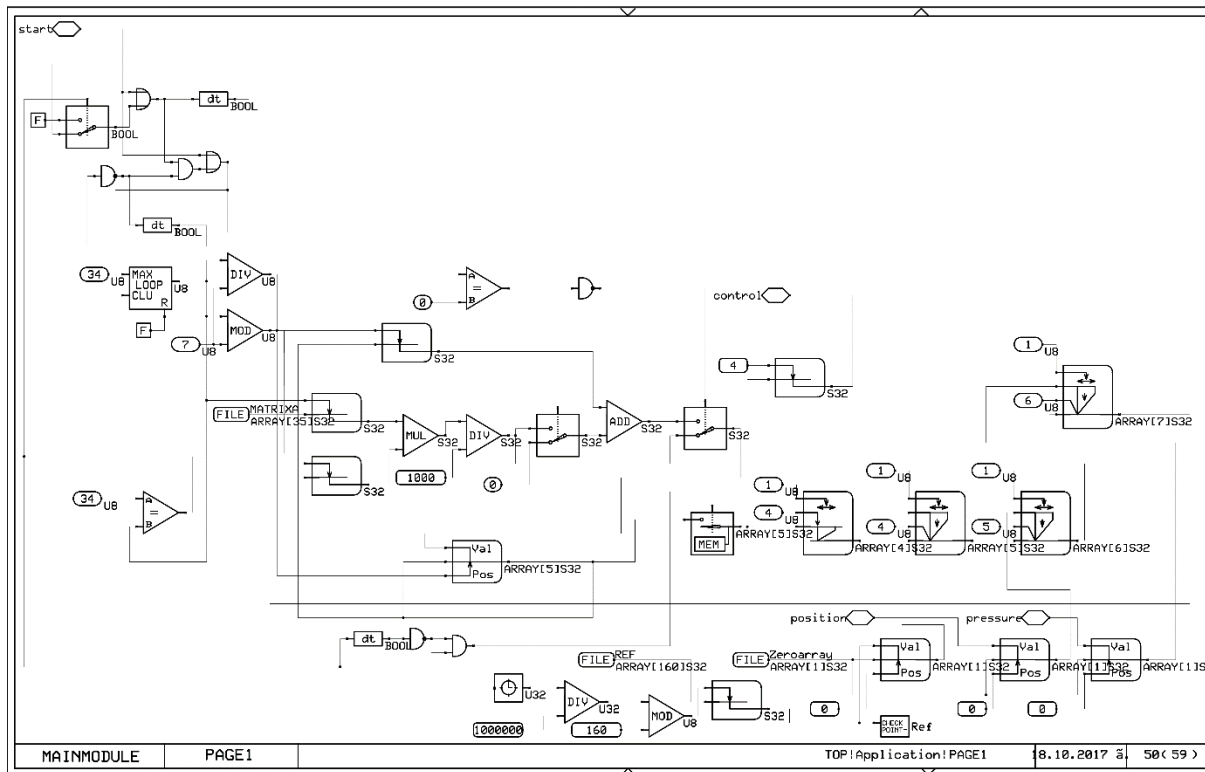
where I_2 is second order unit matrix and matrix D_f is the positive semi-definite solution of Riccati equation

$$AD_f A^T - D_f - AD_f C^T (CDC^T + 10^{-4} I_2)^{-1} CD_f A^T + K_v D_v K_v^T = 0$$

The matrix $D_v = \begin{bmatrix} 108.97 & 0 \\ 0 & 27.44 \end{bmatrix}$ is the variance of noise $v(k)$.

Experimental results

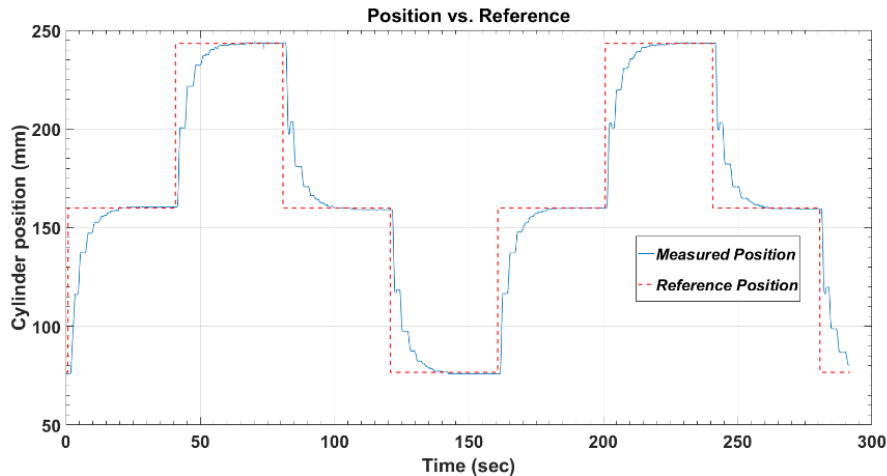
Experiments with the designed LQR regulator require specific implementation technique in target microcontroller MC012-022. The controller interconnection can be represented in equivalent single matrix vector formulas.



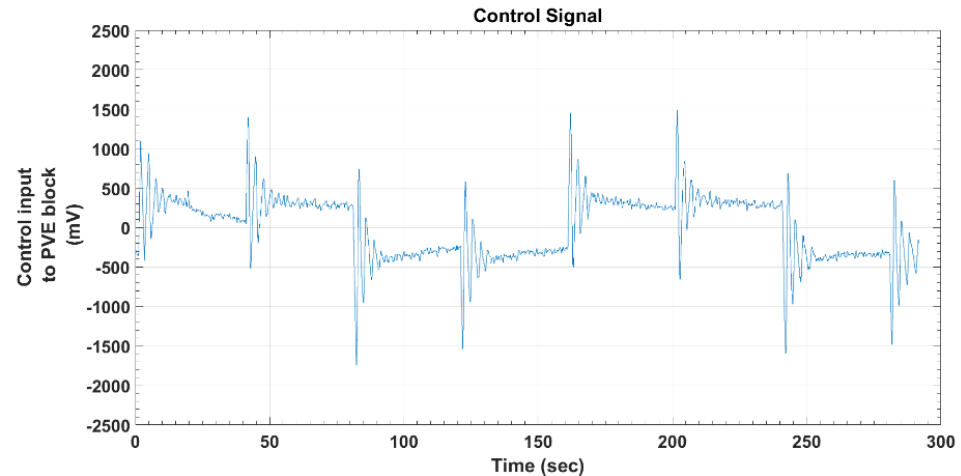
$$\begin{pmatrix} \hat{x}(k+1) \\ x_i(k+1) \\ u(k+1) \end{pmatrix} = \begin{pmatrix} A-CA & 0 & B-CB \\ -T_S C & 1 & 0 \\ K_c & -K_i & 0 \end{pmatrix} \begin{pmatrix} \hat{x}(k) \\ x_i(k) \\ y(k) \end{pmatrix}$$

$$y(k) = \begin{pmatrix} y_{ref}(k) & y_{press}(k) & y_{pos}(k) \end{pmatrix}^T$$

Step response of closed-loop



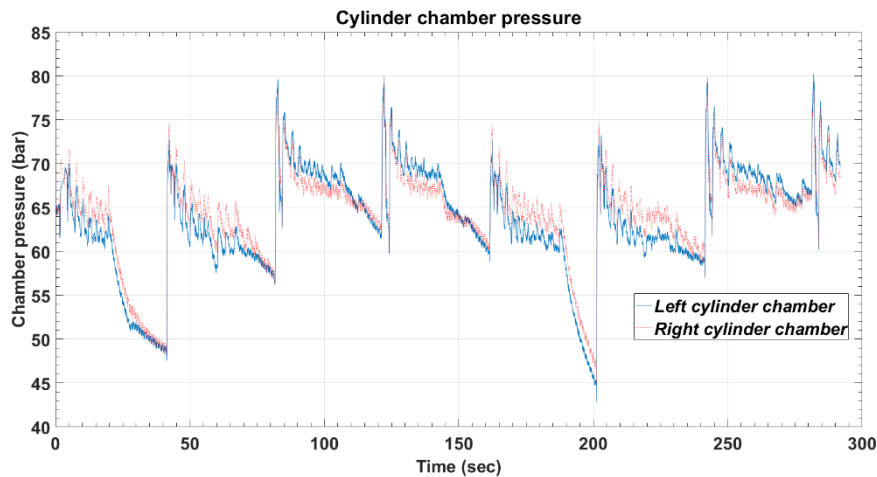
Control signal to PVE block



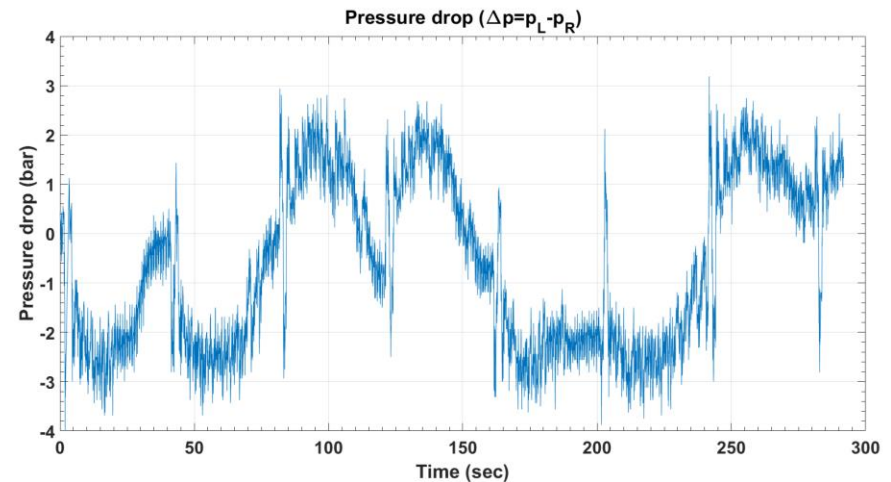
There is zero error in steady state. Transitional processes are of an aperiodic nature, without overshoot. The quality of the transition processes is maintained when cylinder piston moving in both directions.

The control signal approaches its maximum value during the transition process, indicating that the output reacts with the maximum possible performance boost. The noise level in the control signal is low, indicating a high accuracy of the measuring sensor for the cylinder position. This in turn translates as a quality of the closed-loop system.

Cylinder chamber pressure



Pressure drop cylinder chambers



Experimental studies were performed at a fixed setting of the loading pressure (0.5MPa) set by the load system based on a hydraulic block with over-center valves (pos.11, Fig.1). This system makes it possible to realize different pressure loads in the two chambers of the servo-cylinder.

The variable pressure load affects the closed system as a low-frequency output disturbance. The results show the low sensitivity of the system to it. It's an important quality because there is no need to re-set the system at different loads. This insensitivity occurs at the expense of the increased power of the control signal.

- ✓ The main result of the paper is a developed system for **LQG** control of electrohydraulic steering system that is implemented in low speed mobile machines.
- ✓ A two output one input discrete time stochastic plant model is obtained by identification procedure. This model is validated by statistical tests and is used to design of **LQR** controller and **Kalman filter**. The multivariable **LQR** regulator with integral action and **Kalman filter** are designed. Appropriate software which is implemented in 32-bit microcontroller is developed.
- ✓ The results from experiment with developed by authors laboratory setup confirm control system performance. The embedded control system achieves the prescribed requirements: fast transient response without overshooting and static error in whole working range.

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Thank you for your attention!

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